

## Math Problem Solving Phases on Thinking Outside The Box

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**Abstract:** This research resulted problem solving stages/phases for thinking outside the box which was simpler in using the terms. Problem solving phases included exploration, unique idea generated and justification. Exploration phase had two criteria, namely problem exploration and mathematical interpretation. In interpreting mathematical problems, the students used sub-concepts (the prerequisite knowledge) had by them. Review was done on the sub concepts previously acquired which were identified by using two cases: (1) exploration by categorical, namely sub concepts were learned through example; and (2) exploration by compositional, namely sub concepts were learned through the elements/components. Unique idea generated phase contained two criteria, namely conjecture and representation. In making conjecture, solution strategy reviewed was the solution strategy which was different while the justification phase contained a single criterion, namely justification. Justification toward the solution of mathematical problems by students was reviewed through the logic and accuracy in finishing the problems. The process of settlement which was logical and right showed the depth of understanding to the mathematical concepts that had been taught. The abstract should summarize the content of the paper. Try to keep the abstract below 200 words. Do not make references nor display equations in the abstract. The journal will be printed from the same-sized copy prepared by you. Your manuscript should be printed on A4 paper (21.0 cm x 29.7 cm). It is imperative that the margins and style described below be adhered to carefully. This will enable us to keep uniformity in the final printed copies of the Journal. Please keep in mind that the manuscript you prepare will be photographed and printed as it is received. Readability of copy is of paramount importance .

**Keywords:** Phases, Thinking, Outside the Box

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### I. Introduction

Problem solving closely relates to the activity of thinking while the activity of thinking is an activity of the human brain. The human brain is formed by layers which contain networks of neurons which are related one another. The complexity of these networks determine the level of thinking ability from the low order thinking skills, also called the basic thinking skills, to high order thinking skills such as critical, creative, logical, analytical, and reflective thinking ability.

Herrmann (2001) [1] states creative thinking is as "out of the box thinking" or "outside the box thinking". The understanding of the human brain dominance profile leads to the definition of mental "box" as a limitation that people create for themselves. This happens because the brain has a way of working that is efficient. In other words the brain develops repeated thinking patterns every day, so it evolves into a mental "monotonous (default)". Therefore, when brain faces a decision or a challenge, the brain will easily estimate the limits of perspective that can be done. It means mental "monotonous" affects all activities undertaken. In emergency situation, mental "monotonous" may help the brain to get the right and fast decisions, but for some students, the existence of mental "monotonous" sometimes creates a "happy" and "comfortable" zone that leads to a reluctance to upgrade their quality, therefore it is necessary for a person to get out of our own box so his/her quality increases to be the better one.

Thinking outside the box is more widely used to solve social problems such as the brilliant strategy of marketing the product, and only a few are used to solve problems in mathematics. Thinking outside the box is a term used to denote the level of a higher way of thinking, that is, when the quality of the solution to the problem or idea of settlement is substandard and settlement is undiscovered (non-routine).

Thinking outside the box in solving mathematical problems cannot be separated from the creativity, and creativity itself is one piece of self-actualization that is at the highest level of Maslow's hierarchy of human needs. Creativity in mathematics according to Levenson (2013) [2] is marked by several ways, namely (1) using a non-algorithmic way; (2) thinking flexibly and divergently to bring a lot of different strategies for a problem; (3) insightful and unusual solutions.

Polya in How to Solve It (2004) [3] suggests four phases to get the solution of mathematical problems. **The first phase**, understanding the problem, the problem statement must be understood. In this case the students

must read the statement of the problem to see clearly the things that are necessary. **The second phase**, students must see the connection between what to be addressed with the data to get an idea of the solution and make a plan. **The third phase** is implementing the plan; **the fourth phase**, look back, students look back the solutions that have been made and discussed.

Polya's phases are too common for problem solving in thinking outside the box, so this research will generate the phases of problem solving that is simpler to use the term. This research is important for educators to make thinking outside the box as input information about the characteristics of student's thinking, through his behavior that can be observed. In addition, the research of thinking outside the box can be a contribution to enrich the knowledge of the thinking process, especially in the field of mathematics. Problem solving closely relates to the activity of thinking while the activity of thinking is an activity of the human brain. The human brain is formed by layers which contain networks of neurons which are related one another. The complexity of these networks determine the level of thinking ability from the low order thinking skills, also called the basic thinking skills, to high order thinking skills such as critical, creative, logical, analytical, and reflective thinking ability.

## II. Methods

The researcher prepared the instrument in the form of a mathematical problem. The instrument was designed based on the theories that had been arranged. To get the appropriateness and accuracy of instruments, the researcher conducted a direct observation to the prospective research subject. The observation was done for testing the instrument. To know the reading level of instrument, the researcher asked information to the mathematics teachers of junior high schools/senior high school and the mathematics lecturers. This was done to facilitate the next step for the researcher in validation process. In addition, the input from junior high school/senior high school math teachers was used by the researcher as the material to get a picture of how teachers taught the students to solve a mathematical problem. Next the input presented was disaggregated to distinguish the conventional solution and non conventional solution. It was expected later the solution produced by the students was in the form of non-conventional solutions, which was the preferred solution in this research. Meanwhile, to determine the validity level of the instrument, the instrument was validated by two validation experts. The instrument which was validated was given to the research subject to be completed.

In accordance with a predetermined schedule, the researcher gave a mathematical problem to the subject of research to be completed. In this case, the researcher used a think aloud technique. The researcher watched the research subject when solving mathematical problems, recorded all the activities of research subjects by using a digital camera. Therefore, the observations obtained were not only in the form of mathematical problem solution but also in the form of recording moving images (video) of student when doing the math problem. To complete the observed data, the researcher conducted a semi-structured interview to deepen the process of thinking outside the box of student.

At the stage of data analysis, the researcher conducted a series of data analysis stages that included: (1) transcribing verbal data, that was the data obtained from think alouds and semi-structured interview; (2) studying all data both verbal data and field notes; (3) reducing the data to create abstractions; (4) arranging and coding the data; (5) describing and explaining the description of students' thinking outside the box in solving mathematical problems; (6) making the analysis of student's thinking outside the box with reference to the stage of problem solving thinking outside the box; and (7) drawing conclusions about the problem solving stages of student's thinking outside the box in solving mathematical problems.

## III. Result/Finding

This research described the problem solving phases of student's thinking outside the box in solving mathematical problems. The researcher began the research by giving math problems to the student. The student looked embarrassed to speak. The student just read math problems for a while and murmured. The researcher repeatedly asked the student to speak loudly when he did think alouds. The student did the math problem in silence. It needed hard enough effort to make the student revealed his thought. The researcher understood the condition of the student who looked difficult writing down the problem resolution while talking loudly. It was because the student felt nervous and afraid to lose concentration when solving mathematical problems while talking. The researcher gave stimulation to the student by asking a reason for writing the steps to resolve the mathematical problem that had been made. In the early stage of problem solving, the student responded by paraphrasing the problem situation,

*Each tin will be fully charged after being poured  $\frac{1}{5}$ , poured again  $\frac{1}{5}$ ,  
poured again  $\frac{1}{5}$ , poured again  $\frac{1}{5}$  and poured again  $\frac{1}{5}$  on every other  
can. Because it needs 5 cans to be poured into other cans, it means 5 x*

5=25 and it is added with five cans that had been poured into other cans.

The student marked some important sentences (according to the students) as Figure 1. The important sentences among others were: (1) Iswanto only needs 5 tin cans to make other cans fully charged; and (2) the content of each tin can now grows exactly  $\frac{1}{5}$  of the can initially. Students understood the problem by assuming that 1 tin could fulfill 5 other cans. The student did not see 60 liters of soy milk a whole as an important thing to be paid attention. In addition, student also marked the phrase that indicated the objective to be achieved, namely the phrase "Specify many cans before taking!" It means that students clearly understood the objective to be achieved on math problems. At this stage, the researcher named it as the exploration stage.

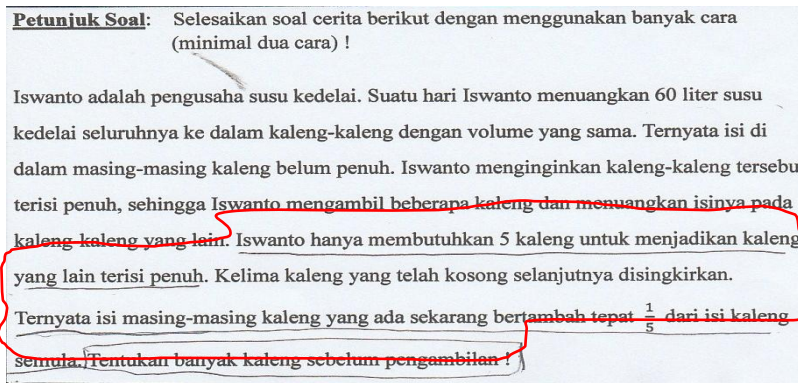


Fig. 1 The student marked some important sentences in a math problem, can be seen in the marker made by the researcher.

In the next stage of problem solving, the student interpreted the sentences that had been marked with the interpretation "5 cans divides soy milk exactly  $\frac{1}{5}$  on the other cans." It was demonstrated by the activity of students in Figure 2. The student added the fractions  $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5} = 1$ . In this case, 1 showed one can of soy milk. Since there were 5 cans, the student wrote back fractional summation process  $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5} = 1$ , thus the need of 5 cans was fulfilled.

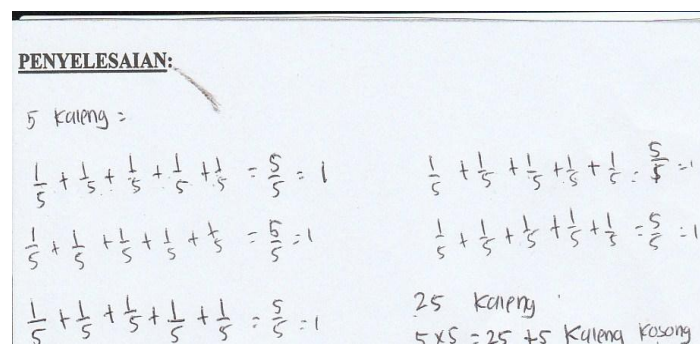


Fig. 2 The student made the first accomplishment of math problem

The number of fractions  $\frac{5}{5}$  showed the amount of soy milk cans after being deducted by 5 cans. Therefore, the final result was 25 cans. The explanation of the student was "  $\frac{1}{5}$  of 5 cans is given to the other cans, of which 1 to 5 tin cans, 1 other tin can is for 5 cans, 1 other tin can is for 5 cans and so on". The student was less able to associate the concept of mathematical problem that had been previously taught like the concept of one variable linear equation. To get the final result, the student added 25 cans and 5 empty cans, so there were 30 cans. At the stage of this problem solving, the researcher named it as the stage of a unique idea formation. The uniqueness could be seen from its completion. The completion of the student was based on the basic concept owned and did not use a settlement that had been taught in previous student.

In the next stage of problem solving, the student developed a solution by providing the second completion as in Figure 3. For the second completion, the student did not make a sound either when writing down the steps to the completion.

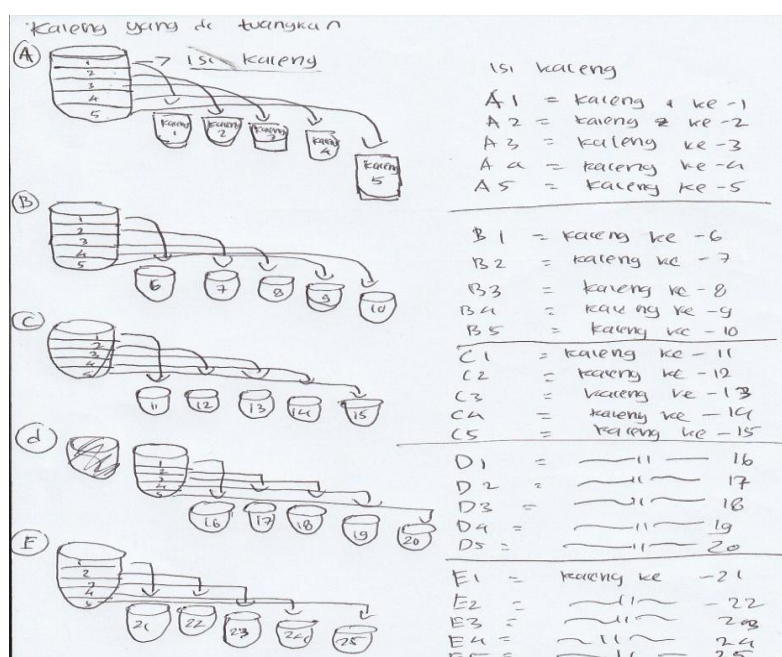


Fig. 3 The second completion of student

The student used an illustration to make the mathematical problem solving. The student named 5 cans as can A, B, C, D and E. The completion ways done by the student were described as follows: (1) the content of can A was poured exactly  $\frac{1}{5}$  in can1 (A1), can 2 (A2), can 3 (A3), can 4 (A4) and can 5 (A5); (2) The content of can B was poured exactly  $\frac{1}{5}$  in can 6 (B1), can 7 (B2), can 8 (B3), can 9 (B4) and can 10 (B5) ; (3) The content of can C was poured exactly  $\frac{1}{5}$  in can 11 (C1), can 12 (C2), can 13 (C3), can 14 (C4) and can 15 (C5) ; (4) The content of can D was poured exactly  $\frac{1}{5}$  in can 16 (D1), can 17 (D2), can 18 (D3), can 19 (D4) and can 20 (D5) ; and (5) The content of can E was poured exactly  $\frac{1}{5}$  in can 21 (E1), can 22 (E2), can 23 (E3), can 24 (E4) and can 25 (E5). Thus, the result was 25 cans. Furthermore, the student added 25 cans with five empty cans as in Figure 4. The student wrote "25 cans + empty can A + empty can B + empty can C + empty can D + empty can E = 30 cans before taking". However, the student was not able to check back the answer generated. It could be seen in graffiti under a long horizontal line. At this solving problem stage, the researcher named the justification stage.

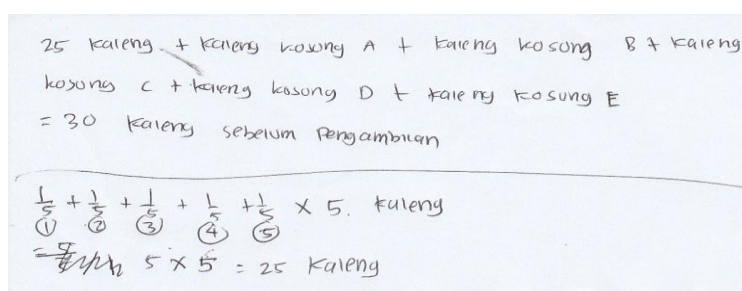


Fig. 4 The student added 25 cans and five empty cans

#### IV. Discussion And Conclusion

Mathematical problem given on the subject of the research consisted of mathematical language that serves as a means of mathematical idea formation. The math problem allowed the research subject to manage and transform the problem context situation in the more abstract symbolic statement in the subject's mind. The math problem also motivated the research subject to manage and integrate mathematical thinking and conceptual elements for the mathematical idea formation.

The math problem was solved by the research subject using the way of thinking that was not limited by the conventional way. In this case the research subject was said using thinking outside the box. This way of thinking could be seen from the way of the research subject looked at the structure of matter. The research subject only focused on a few sentences which were considered important by the research subject by underlining the sentences. The students looked at the situation gradually, imagined step-by-step the situation depiction. Along with the designation by the research subject, in subject's mind there was also the visualization process, namely the depiction of the problem situation based on the language content in the mathematical sentences that had been marked. To get a justification for the visualization result, the research subject needed to find clear and complete information. Therefore, the research subject explored the mathematical problem by describing the information known on the mathematical problem and setting the goal to be achieved. Based on this information, the research subject activated the relevant prior mathematical knowledge to select and set a mathematical concept that would be used.

The theory of Constructive Perception (Solso, 2007) [4] states that people "construct" the perception of stimuli by actively selecting and combining the sensation of memory. Based on the theory of constructive perception, to get an understanding, the research subject read the math problem carefully. The research subject also created marks in a problem statement considered important. It showed that the research subject chose stimuli in order to obtain a sensation (description of problem situation). Moreover, the marker on the problem statement marked by the research subject was how the subject focused. The attention focus of the research subject was the perceptual processing limit to produce a response. The response could arise if the research subject's attention to some important sentences showed meaningfulness.

There are three things related to the knowledge of mathematics, namely the procedure and operation of mathematics, mathematical concepts and mathematical ideas (Kinard, 2008) [5]. Mathematical operation is the process of managing and manipulating mathematical information with meaningful ways that support and build the mathematical ideas and concepts. Mathematical concept is theoretical, systemic, and generative while the idea of mathematics is derived from one or more of conceptual understanding, the establishment of relation between the conceptual understanding and the formation of new ideas or applications. Students who are able to think outside the box are the students who are able to perform mathematical operations correctly to obtain different completion ideas. Different ideas which are generated use self-measurement. In this research, the research subject used the logical power to get a precise mathematical problem solving.

Students who have a high power of reason are able to think tightly / stiffly/ regularly. It is identified by the sharpness of focus and perceptions so that they can do something in problem solving. Mathematical reasoning is a form of symbolic thought (Werner and Kaplan, 1984: 12) [6]. Symbol is a tool used in human cognitive construction. It means that the nature of the symbol presence is as a representation function while the representation of mathematical activity is inseparable from the use of symbol. Math activities generate meaning based on the pattern and relationship through abstraction. Math activities develop and define the knowledge of mathematics through the inquiry process. Therefore, the target of learning mathematics is the accuracy of method, tools and conceptual principles of mathematical knowledge based on cognitive processing.

The mathematical problem solving is done through several stages or phases of problem solving as the following table.

**Table 1.** Phases of mathematical problem solving using thinking outside the box

Phase	Criteria	Student's Activity
Exploration	<ol style="list-style-type: none"> <li>1. Problem exploration</li> <li>2. Mathematical Interpretation</li> </ol>	<ul style="list-style-type: none"> <li>• Formulating information on the problem (what is known?) And objectives (what is being asked?)</li> <li>• Making a picture of the situation based on the information of math problem</li> <li>• Using the appropriate notation</li> <li>• Formulating a mathematical model based on the information and situation of the problem in a math problem</li> </ul>
Unique idea generated	<ol style="list-style-type: none"> <li>1. Conjecture</li> <li>2. Representation</li> </ol>	<ul style="list-style-type: none"> <li>• Outlining the problem, finding a core element of the problem</li> <li>• Pursing the matter and trying to identify the stages of strategy</li> <li>• Associating a problem situation with a concept</li> <li>• Finding the problem analogy, and trying to use a similar solution strategy</li> <li>• Creating concept construction</li> <li>• Designing a solution plan</li> <li>• Choosing a strategy to prove the solution allegation</li> <li>• Implementing solution strategy</li> <li>• Applying procedural to continue the completion of mathematical model that has been formulated</li> </ul>

		<ul style="list-style-type: none"><li>• if there is a deadlock, the students check information on the problem, then create the mathematical model</li></ul>
Justification	Justification	<ul style="list-style-type: none"><li>• Finding / getting the solution of mathematical problems</li><li>• Re-checking the resulted solution. If the solution is wrong, the improvement is re-checked : (1) the interpretation of the problem so that the process with a new strategy is produced; (2) procedural or (3) Conceptual</li><li>• Making excuses of the answer to prove the correctness of the resulted solution</li><li>• Finding alternative ways to solve the problem</li></ul>

### **Acknowledgements**

This research was conducted in the context of the completion of a dissertation under the guidance of dissertation supervisor. Therefore, the researcher would like to thank to the dissertation supervisor on conceptual contribution so that this research went easily and smoothly.

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